

Exercise 31

If $f(x) = \ln(x + \ln x)$, find $f'(1)$.

Solution

Take the derivative of the function with respect to x by using the chain rule.

$$\begin{aligned} f'(x) &= \frac{d}{dx} \ln(x + \ln x) \\ &= \frac{1}{x + \ln x} \cdot \frac{d}{dx} (x + \ln x) \\ &= \frac{1}{x + \ln x} \cdot \left(1 + \frac{1}{x}\right) \\ &= \frac{1}{x + \ln x} \cdot \left(\frac{x + 1}{x}\right) \\ &= \frac{x + 1}{x(x + \ln x)} \end{aligned}$$

Now set $x = 1$ to get $f'(1)$.

$$f'(1) = \frac{1 + 1}{1(1 + \ln 1)} = \frac{2}{1(1 + 0)} = 2$$